Differential Geometry and Meshing for Scientific Computing, 6.0 credits

Differential Geometry and Meshing for Scientific Computing, 6.0 hp

Third-cycle education course

6FMAI33

Dept of Mathematics

Valid from: Second half-year 2025

Approved by
Head of Department

Approved
2025-10-09

Registration number

Entry requirements

Students should be acquainted with linear algebra, multivariable calculus, numerical analysis (interpolation and quadrature in particular), and computer programming. Any gaps in the background can be closed by students at the beginning of the course by intensive self-studies (possibly with the help of the examiner).

Learning outcomes

After this course students should be able to:

- Describe how curves and surfaces can be represented using discrete methods
- Work with generalized coordinate systems and perform Calculus within them
- Explain key concepts and algorithms used in mesh generation
- Understand and apply practical aspects of curve approximation and mesh generation in computational settings

Contents

Curves (in 2D) and surfaces (in 3D) can be represented using well-established approximation techniques like Bézier curves, polynomial interpolants, and splines. Understanding and evaluating the quality of these discretizations involves key geometric concepts like curvature, which can be studied from both computational and mathematical perspectives.
This analysis draws on tools from multivariable calculus and linear algebra. Exploring the interplay between continuous and discrete differential geometry enhances understanding in both areas and leads to the development of practical algorithms for real-world applications.
There are many approaches to mesh generation—that is, decomposing a complex domain into smaller, more tractable elements. Choosing the “best” strategy is still an active area of research. This course will cover both classical and modern mesh generation techniques, including conformal mappings, Delaunay triangulation, cubing the sphere, and templating. We will also discuss metrics for evaluating mesh quality and examine how mesh quality impacts the performance of discrete differential geometry tools.

Educational methods

Students should attend all the lectures and satisfactorily complete the assigned theoretical and computer exercises. If a student cannot participate in a course session then they are responsible to review the background material of said session. They would then meet with the examiner to discuss the topics and (possibly) be asked to demonstrate proficiency in the topics covered in any missed sessions.
More details on this are provided in the Examination section.

Examination

Course examination consists of four components:

- **Attendance**: Students participate in and contribute to the course sessions. If a student misses a course session, they are responsible to review the background material of said session. Students would then meet with the examiner to discuss the topics and (possibly) be asked to demonstrate proficiency in the topics covered in any missed sessions.
- **Lecture**: Each student will select (or be assigned) a topic. They must then read provided background literature and develop a presentation ≈45 minutes in length to present the topic to the other students and the examiner.
- **Theory exercises**: Throughout the course there will be theoretical exercises that the students must complete and submit to the examiner. Sufficiently correct solutions to these exercises are required. Some of the theoretical exercises may be presented and discussed during the course sessions.
- **Computer exercises**: Several computer programming exercise sets will be given to reinforce the topics covered during the sessions. Sufficiently correct and tested implementations for these computer based exercises are required.

Grading

Two-grade scale

Course literature

The course is based on book chapters from several sources as well as many classic and modern journal articles on the topics of discrete differential geometry and meshing. These articles will be provided by the examiner and are too numerous to explicitly name. The books upon which the course is built are the following:

- *Handbook of grid generation*. Thompson, Joe F., Bharat K. Soni, and Nigel P. Weatherill, eds. CRC press, 1998. https://doi.org/10.1201/9781420050349
- *Numerical grid generation: Foundations and applications*. Thompson, Joe F., Zahir UA Warsi, and C. Wayne Mastin. Elsevier North-Holland, Inc., 1985.
- *Shape interrogation for computer aided design and manufacturing*. Patrikalakis, Nicholas M., and Takashi Maekawa. Vol. 15. Heidelberg: Springer, 2002. http://deslab.mit.edu/DesignLab/tmaekawa/book/mathe.pdf

Supplemental information is drawn from

- *Differential geometry of curves and surfaces*. Banchoff, Thomas F., and Stephen Lovett. Chapman and Hall/CRC, 2022. https://doi.org/10.1201/9781003295341