Applied Topology, Topological Data analysis, 6.0 credits

Tillämpad topologi, Topologisk dataanalys, 6.0 hp

Third-cycle education course

6FMAI28

Department of Mathematics

Valid from: First half-year 2024

Approved by
Head of Department

Approved
2024-03-22

Registration number
DNR MAI-2024-00038

Entry requirements

Linear Algebra (1st course) and basic concepts in the topology of Rn.

Learning outcomes

Topological Data Analysis has become an important tool in machine learning and artificial intelligence. The course will provide the needed concepts, methods and tools in homology (persistent homology and persistent diagrams) to have efficient methods to study data clouds and other objects in data analysis.
After the course the student should be acquainted with the basic concepts in topological complexes, homology and persistent homology, and its use in data analysis and other subjects, Specifically, they should be able to approximate data clouds and other geometrical objects by topological complexes and filtrations and, calculate the persistent homology of these mathematical objects and the persistent diagrams. After the course the participants should be able to give reliable interpretations of the topological invariants in applications in technology and humanities. Besides, they should be able to use software packages to calculate homology of complexes and persistent diagrams, and to carry out analysis data with these methods.

Contents

1. Homology: CW-complexes and simplicial complexes. Construction of complexes from data: Check and Rips complexes. Filtrations and Discrete Morse Theory. Complex homology. Methods to calculate homology
2. Persistent Homology: Definition of persistent homology, persistent diagrams. Calculations of persistent diagrams. Stability Theorem
3. TDA and Other Applications: What is TDA, examples and applications. Applications to discrete optimization. Construction of complexes from data. Digital topology. Spaces of configurations.

Educational methods

Lectures on location.
Virtual Teaching:
1. Digital Tests and Hand-in exercises
2. Online meetings (e.g. via ZOOM) student-student and teacher-student.
3. Laboratories on the virtual classroom.
4. Didactical materials online.

Examination

Hand-in Exercises and presentations (virtual).

Grading

Two-grade scale

Course literature

Main References:

1. G. Carlsson, *Persistent Homology and Applied Topology*, ArXiv 2020

(https://doi.org/10.48550/arXiv.2004.00738)

1. A. Rosenfeld, *Digital Topology*, The American Mathematical Monthly , Oct., 1979, Vol. 86, No. 8 (Oct., 1979), pp. 621-630.
2. R.Ghrist, Elementary Applied Topology, ed. 1.0, Createspace, 2014. www2.math.upenn.edu/~ghrist/notes.html. Capter 1.
3. A. Zomorodian and G. Carlsson, *Computing Persistent Homology*, Discrete and Computational Geometry 33 (2005) 249-274.

Other Literature:

1. Alexander D. Smith, Pawel Dlotkoy and Victor M. Zavala, *Topological Data Analysis: Concepts, Computation, and Applications in Chemical Engineering*, ArXiv Nov. 2020.
2. Ziga Virk, Introduction to Persistent Homology, Univerza v Ljubljani (Universidad de Liubliana, Eslovenia), 2022.
3. G. Carlsson, Persistent Topology and the Analysis of High Dimensional Data, University of Chicago, 2005 (pdf-file with Lecture Notes by G. Karlsson)
4. H. Edelsbrunner and J. Harer, *Persistent Homology—a Survey*, *Contemporary mathematics*, *453*, 257-282.
5. Herbert Edelsbrunner and Dmitriy Morozov, Persistent Homology: Theory and Practice, Proceedings of the European Congress of Mathematics, 2012.
6. Robert Ghrist *Barcodes: the persistent topology of data*. Bulletin of the American Mathematical Society (New Series) 45, 1 (2008), 61–75.
7. Magnus Bakke Botnan, Topological Data Analysis, Lecture Notes, Spring 2020
8. E. Carlsson, J. G. Carlsson and S. Sweitzer, Applying Topological Data Analysis to local search problems. Foundations of Data Science, 4 (2022) 563-579.

General information

The Course is taught as a Blended Course.