Riemann Surfaces, 8.0 credits

Riemannytor, 8.0 hp

Third-cycle education course

6FMAI21

Dept of Mathematics

Valid from: First half-year 2024

Approved by
Head of Department

Approved

Registration number

Entry requirements

Courses in geometry, algebra and complex analysis.

Contents

This course is an introduction to Riemann surfaces with an algebraic and geometric viewpoint as the subtitle of the book followed by the course says. In the course we will work with
1. Analytical functions between Riemann surfaces are branched coverings. In fact one can see a Riemann surface as a branched covering of the Riemann Sphere (earlier work of Schwarz, Hurwitz, Weierstrass, Clebsch, Klein, and many more).
2. The group of meromorphic functions on a Riemann surface is (functorially) the group of fractional functions of a projective smooth curve. So compact Riemann surfaces are projective complex curves (earlier work of Schottky, Wiman, Torelli, Fricke, and many more).
3. Finally any R. S. is the quotient of either the sphere, the plane or the hyperbolic plane by a discrete subgroup of the corresponding group of motions (Poincar, Koebe).
The **flexible** contents:
1. Prerequisites. Conformal and meromorphic functions: zeros, poles, Cauchy Integral Formula, Liouville's Th., series, Maximum Modulus Principle. Coverings and Fundamental Groups: Surfaces, coverings, universal coverings, fundamental groups and groups of deck-transformations of coverings, monodromies.
2. Riemann Sphere and Möbius Transformations. Meromorphic and rational functions. The group PSL(2, C).
3. Elliptic Functions and Tori. Weierstrass p-function. Topology of elliptic functions and tori.
4. Riemann Surfaces. Meromorphic functions and germs. Connected components in the space of germs of meromorphic functions. The space of tori.
5. Fuchsian Groups. Discrete subgroups and discontinuous actions. Fundamental regions and Riemann surfaces. Uniformization Th.

Educational methods

Lectures and seminars.

Examination

To pass the course a student should** **solve most of the proposed and give a (50 minutes) seminar on a topic related to the contents of the course.

Grading

Two-grade scale

Course literature

Main Reference: G. A. Jones & D. Singerman, *Complex Functions, An Algebraic and Geometric Viewpoint*. Cambridge Univ. Press, Cambridge, 1988
Other Literature:
1. F. Beardon, *The Geometry of Discrete Groups*. Graduate Texts in Maths. 91, Springer, Berlin, New York, 1983
2. W. S. Massey, *A Basic Course in algebraic Topology*. Graduate Texts in Maths.127, Springer, Berlin, New York, 1991
3. H. M. Farkas & I. Kra, *Riemann Surfaces*. Graduate Texts in Maths. 71, Springer, Berlin, New York, 1992
4. R. D. M. Accola, *Topics in the Theory of Remann Surfaces*. Springer, Berlin, New York, 1994
5. R. Miranda, *Algebraic Curves and Riemann Surfaces*. Graduate Studies in Maths. 5, AMS, Providence, 1995
6. J. E. Marsden & M. J. Hoffman *Basic Complex Analysis*. W.H. Freeman Berlin, New York, 1999
7. A. F. Beardon, *Algebra and Topology*. Cambridge Univ. Press, Cambridge, 2005
8. E. Girondo & G. Gonza{'a}lez-Diez, *Introduction to Compact Riemann Surfaces and Dessins d'Enfants*. Cambridge Univ. Press, Cambridge, 2012
9. R. Cavalieri & E. Miles, *Riemann Surfaces and Algebraic Curves*. Cambridge Univ. Press, Cambridge, 2016